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Bridge deterioration modeling using semi-Markov theory

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ABSTRACT: The most widely used deterioration model today is one that is based on the Markov chain theory. In this paper, the unnecessarily restrictive requirement for exponential or geometric holding times of the Markov process is relaxed in a more general stochastic model called semi-Markov process. Examples using NBI data (USA) are provided which illustrate this new approach.

1 INTRODUCTION

The reliability of a bridge declines with time due to the degradation of the material and the increase in vehicular load. To effectively manage an existing bridge stock requires that the bridges be monitored throughout their life spans.

Bridge deterioration modeling is thus a very important activity in bridge management system (BMS) development. It involves establishing a relationship between the bridge performance and time. This relationship may be assumed to be deterministic or stochastic in nature. A *deterministic deterioration model* assumes that future bridge performance (or its expected value) is known with certainty. In this case, the performance-time relationship is described by a mathematical equation relating either the performance or expected performance with time. A commonly used deterministic model is the regression function obtained by doing a regression analysis on historical bridge data (Veshosky et al 1994).

A *stochastic model*, on the other hand, treats the deterioration process as a stochastic process. The state-of-the-art stochastic model has been based on the Markov chain theory (Jiang & Sinha 1990, Cesare et al 1992). In the Markov-chain deterioration model, the performance level is specified as discrete *states*. The performance of the bridges changes from one state to another in accordance with a set of transition probabilities p_{ij}. p_{ij} is defined as the probability for the bridge to move from state i to state j in one step; which may be one year; or two years etc.

The Markov-chain bridge deterioration model often assumes that a bridge can either remain in the current state or deteriorate to the next lower state in one step. Also, the worst state N in a state space of $\{1, 2, ..., N\}$ is considered an absorbing state; which means that once the process enters the state it will never leave it. The stochastic nature of the deterioration process is thus characterized by the transition probability matrix of this format:

$$\boldsymbol{P} = \begin{bmatrix} p_1 & 1-p_1 & \cdots & 0 & 0 \\ 0 & p_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & p_{M-1} & 1-p_{M-1} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad (1)$$

where p_i, $i = 1, 2, ..., N$ represents the probability of remaining in the ith state in the next step. Notice that p_N is equated to 1 since N is an absorbing state.

There is now wide acceptance of the use of Markov-chain deterioration models but it is important to investigate the validity of the Markov chain assumptions in bridge deterioration modeling.

First, the Markov chain theory stipulates that the transition probabilities depend only on the current states and not on how the current states had been reached ("Markov property"). Second, the transition probabilities are constant over time - a property called "time homogeneity." Implicit in the homogeneity property of the Markov process is the requirement of exponential (for continuous time) or geometric (for discrete time) distributed holding times. *Holding time* is the amount of time a process sojourns in one state before moving to another. The exponential and geometric distributions possess the so-called "memoryless" property. This property, used in the context of bridge deterioration, suggests that the probability for a bridge to move from its current state to another more deteriorated state does not depend on how long it has been in the current state. Regardless of the validity of this assumption, the requirement of exponential or geometric distributions is unnecessarily restrictive. This requirement for exponential or geometric holding time in the Markov process is relaxed in a more general stochastic model called semi-Markov process. This paper investigates important aspects of bridge deterioration modeling using the semi-Markov theory.

2 SEMI-MARKOV BRIDGE DETERIORATION MODEL

2.1 Semi-Markov Process

A semi-Markov process is a class of stochastic process which moves from one state to another with the successive states visited forming a Markov chain; and that the process stays in a particular state for a random length of time the distribution of which depends on the state and on the next state to be visited (Ross 1970).

To better understand this concept, the semi-Markov process could be conceived as a stochastic process governed by two different and independent random-generating mechanisms. When the process enters any state i, the probability of it moving to the state j in the next transition is specified by the transition probability p'_{ij}. Once the successor state was determined and prior to the transition the process stays in the current state i for a duration T_{ij} which is dictated by the *holding time* probability density function $h_{ij}(t)$. For discrete-time models $h_{ij}(m)$ will be used in place of $h_{ij}(t)$.

From the above discussions, it is obvious that one way to describe a semi-Markov process would be to use the transition matrix $\boldsymbol{P'}$ and the holding time matrix $\boldsymbol{H}(t)$:

$$\boldsymbol{P'} = \{ p'_{ij} \} \qquad i = 1, 2, ..., N; j = 1, 2, ..., N$$
$$\boldsymbol{H}(t) = \{ h_{ij}(t) \} \qquad t \geq 0 \tag{2}$$

2.2 Bridge Deterioration as a Semi-Markov Process

We now explore how to model bridge deterioration as a semi-Markov process. A sample function of the deterioration process (under no human intervention) is shown in Fig. 1. It shows that a bridge would begin with the best state, remain there for a period of time before degrading to the next worse state; so on and so forth. Notice that the sample function is a monotone non-increasing function. Under this condition, $p'_{ij} = 1$ for $j = i+1$ and $p'_{ij} = 0$ for all other values of j except the final state, N. It is assumed that the last state N is an absorbing state and thus p'_{NN} is 1. The transition matrix is accordingly given as

$$\boldsymbol{P'} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{3}$$

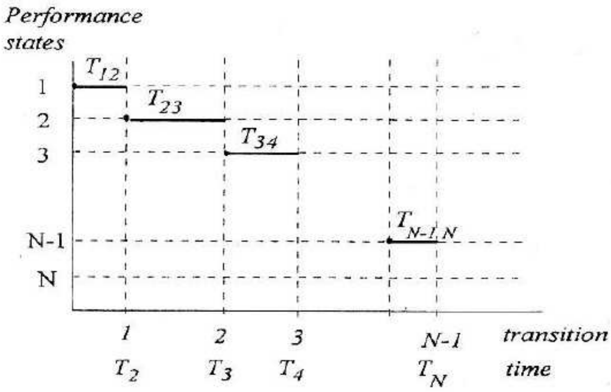

Fig. 1 A Sample Function of the Deterioration Process

The modeling of bridge deterioration process (under no human intervention) using semi-Markov process is thus reduced to one of determining the holding time distribution $h_{ij}(t)$. Chapter 4 will discuss a procedure to derive the distributions from historical bridge data.

In the semi-Markov process formulation, a Markov chain process could be regarded as a special case of the semi-Markov process by requiring real transition between two different states and specifying both the transition probability p'_{ij} and the holding time it takes for such transition. As an illustration, the Markov chain deterioration process in Eq. (1) can also be represented by Eq. (4) and Eq. (5) (Ng 1996).

$$P' = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{4}$$

$$H(m) = \begin{bmatrix} 0 & p_1^{m-1}(1-p_1) & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & p_{N-1}^{m-1}(1-p_{N-1}) \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{5}$$

The holding times $h_{ii+1}(m)$, $i = 1, \ldots, N$-1 are geometric distributed.

3 PERFORMANCE PREDICTION AND SEMI-MARKOV DECISION PROCESS

In many existing bridge management system (BMS) applications, for example, Pontis (Golabi 1993), bridge deterioration models are used in:

i. the prediction of future performance
"Given the current status of the bridges in the network what will be future needs in, say, five years' time?" To answer this management question requires the prediction of future bridge performance for the whole network.
ii. the construction of the decision model
"What is the optimal improvement policy for the entire network which would minimize the bridge costs?" A *policy* is a rule which prescribes decisions corresponding to each performance state of the bridge. Mathematically it is a relationship between the state space of the bridge and the decision or action alternatives:

$$\Re = \{ A_1, \ldots, A_i, \ldots, A_N \}^{\mathrm{T}} \tag{6}$$

where A_i is the decision/action prescribed for state i.

3.1 Traditional Markovian Models

With deterministic deterioration models such as that using the regression functions, bridge performance prediction and decision model are rather straight forward and will not be further discussed in this paper. The reader is referred to Jiang & Sinha (1993).

For stochastic deterioration models, future performance is described by a *state probability vector* which specifies the probability of the bridge being in each performance state. For Markov-chain deterioration model, performance of the bridge network at a future time m is calculated by using this relationship:

$$\pi(m) = \pi(0) \times \{ p_{ij} \}^m \tag{7}$$

where $\pi(m)$ is the state probability vector at any time m and $\pi(0)$ is the initial state probability vector. Eq. (7) says that the probability distribution of the state in m years' time is the matrix product of the initial distribution and the transition matrix raised to the power m. The matrix $\{ p_{ij} \}^m$ is called the m-step transition matrix or *interval probability matrix*.

In determining the optimum policy for bridge improvement the traditional approach uses the concept of Markovian decision process (Golabi 1993). The decision process assumes that at every fixed interval an action must be chosen. The action belongs to a finite set of feasible actions for that state; which include the do-nothing action, various degrees of rehabilitation and replacement. If the process is in state i and action k is chosen, then

i. an immediate expected cost of ${}^kC_i$ is incurred;
ii. the process next evolves to another state in accordance to the transition probability ${}^kp_{ij}$.

The decision process is therefore a result of human intervention (in the form of decisions) and deterioration process. The optimality equation is given as

$$v_i(n+1) = \min_k \left[{}^kC_i + \eta \sum_{j=1}^{N} {}^k p_{ij}\, v_j(n) \right] \tag{8}$$

in which $v_i(n+1)$ is defined as the present value of the costs from the remaining n+1 time periods if the system is now in state i and the optimal selection of alternatives has been performed at each stage through stage n. η is the discount factor so that the present value of one unit of cost n periods in the future is η^n. In Eq. (8), the first term in the square brackets is the cost consequential to the first "transition" or rather, the first "period"; the second term is the expected total discounted cost of the system evolving over the remaining n periods.

For infinite planning horizon where n is very large, $\lim_{n\to\infty} v_i(n) = v_i$. Eq. (8) is thus re-written as

$$v_i = \min_k \left[{}^kC_i + \eta \sum_{j=1}^{N} {}^k p_{ij}\, v_j \right] \qquad (9)$$

3.2 Proposed Semi-Markov Model

A complete formulation of the semi-Markov model analogous to that of the Markov model have been presented by Howard (1971). His formulae have been adapted here to suit the bridge deterioration process.

For performance prediction (under no human intervention), an equation analogous to that for the Markov chain is given in Eq. (10).

$$\pi(t) = \pi(0) \times \{\ \psi_{ij}(t)\ \} \qquad (10)$$

$\{\ \psi_{ij}(t)\ \}$ is the interval probability matrix much like $\{\ p_{ij}\ \}^m$ in Eq. (7). $\psi_{ij}(t)$ is the probability that the process will occupy state j at time t given that it entered state i at time zero. Now, at time t after entering state i the process would either stay in the current state i or move to a lower state j. Consider τ as the time for the first real transition to occur. If the process has moved to a lower state, then $\tau < t$. In this event, after the process has moved to the next lower state at time τ, it moves to state j in the remaining time $(t-\tau)$. The interval probability is thus given as

$$\psi_{ij}(t) = \int_0^t h_{ii+1}(\tau)\psi_{i+1j}(t-\tau)d\tau, \quad j \neq i \qquad (11)$$

If the process has remained in state i throughout the time t then $\tau > t$. The interval probability is thus the complementary of the probability that the process will move out of the current state (viz., to move to the next worse state) within time t.

$$\psi_{ij}(t) = 1 - {}^{\leq}h_{ii+1}(t), \qquad j = i \qquad (12)$$

where the notation ${}^{\leq}h_{ii+1}(t)$ is used to denote the cumulative distribution function of the holding time.

For determining optimum improvement policy, the Markovian decision process can be generalized to a semi-Markov decision process by (Puterman 1994):

i. allowing or requiring the bridge manager to choose an action whenever the system state changes, and not at fixed intervals;
ii. modeling the system evolution in continuous time and not in discrete time;
iii. allowing the holding time at a particular state to follow an arbitrary probability distribution rather than a geometric distribution.

Whenever the system enters a state i, its successor state j is selected according to the transition probability p'_{ij}. The duration of staying in state i before moving to another state j is dictated by the holding time density function $h_{ij}(t)$. While the process occupies state i, it lays out maintenance expenses at a rate $M_{ij}(u)$ for a duration u after entering state i. Further, when the process leaves a state i to enter state j, there will be a lump sum of cost associated, C_{ij}. Now in a semi-Markov decision process, whenever the system enters a state the probability and cost function governing departure from it are not fixed but dependent on the action/alternative selected. Thus, associated with the k^{th} alternative action in state i will be transition probability, ${}^kp'_{ij}$, ${}^kh_{ij}(t)$, ${}^kM_{ij}(u)$, ${}^kC_{ij}$.

Define a discount rate $\alpha > 0$ so that a unit quantity of money at time t in the future has a value of $e^{-\alpha t}$ today. For an infinite planning horizon the optimality equation is given by Howard (1971) as

$$v_i(\alpha) = r_i(\alpha) + \sum_{j=1}^{N} p'_{ij} \int_0^\infty e^{-\alpha\tau} h_{ij}(\tau)d\tau\ v_j(\alpha) \quad (13)$$

where $v_i(\alpha)$ is the expected total cost when the system just enters state i; corresponding to a rate of α. $r_i(\alpha)$ is the cost incurred after the first transition and is given by

$$r_i(\alpha) = \sum_{j=1}^{N} p'_{ij} \int_0^\infty h_{ij}(\tau) \left[\int_0^\tau e^{-\alpha u}\ M_{ij}(u)du + e^{-\alpha\tau} C_{ij} \right] d\tau \qquad (14)$$

The optimal policy could be obtained by seeking a solution to the optimality equation, Eq. (13). An algorithm known as the 'Policy Improvement' method has been proposed by Howard (1971) for its solution.

The main strength of the semi-Markovian decision processes over the Markov analog lies in the use of random holding times. By treating the holding times as random rather than as fixed regular intervals the semi-Markov decision processes could incorporate the "time-factor" into the model. One example is the consideration of maintenance cost which is inherently time-dependent; another is the consideration of the time needed to effect an improvement of states. In the Markovian decision process, the improvement action is assumed to take effect immediately.

However, due to the absence of "memoryless" assumption in the holding times, it is necessary to assume that all intervening actions must take place at the points in time when the state changes.

4 DERIVATION OF HOLDING TIME DISTRIBUTION

The above application depend on the ability to derive the holding time distributions pertaining to each pair of adjacent states. Consider Fig. 1, if the time to reach each state i (from state 1), viz., T_i is known probabilistically then it may be possible to determine the distributions for holding times T_{ij} by taking the difference of T_i and T_j. First is the issue of finding the probability distributions of T_i. Second is the issue of deriving the holding time distributions from the distributions of T_i. Each of these two issues will be discussed in the following.

4.1 Time to state i T_i

The time to reach a state T_i could be conceived as the time to failure or *failure time*. "failure" is used here to denote a distinct event, in this case, the reaching of the performance level represented by state i. The study of failure time has been the subject of a statistical analysis called survival analysis (Lawless 1990).

In traditional applications, *failure time* data is obtained from *life testing* (in industrial applications) or *clinical trials* (in medical applications). In either cases, a specific number of the subjects of interest are observed for a period of time to obtain their individual times to failure (or time to the relapse of a certain decease). It is not uncommon to find that some of these items on test have been lost to follow up the study or have continued to survive at the end of the study period. As a result, the failure times of these subjects are not observed. The observations are said to be *censored*. Uncensored observations of failure times are called *complete* observations.

Bridges are not subject to life testing for obvious reasons. A procedure proposed by Ng and Moses (1996) use bridge inventory data for survival analysis. For this purpose, two data elements in any bridge inventory data are of paramount importance: the current performance level and the corresponding age when the performance is observed. The data is treated as if obtained from a life test in which the construction of a new bridge is regarded as an entry of the bridge in the life test. This is equivalent to having all the bridges to start simultaneously in the "life test" but to terminate the study randomly by the observation time. In the context of life testing, the time of observation for each bridge can be viewed as a censoring time. It is at the time of observation that one would discern if there is a complete or censored observation.

If the bridge performance equals the state defined as failure there is a *complete* observation. If instead it was found, at the time of observation, that a bridge had not reached the performance state then the observation is considered *right-censored*. This observation though incomplete is useful for it indicates that the failure time of the bridge goes beyond its present age. Further, if a bridge had already surpassed the state at the time of censoring the observation is *left-censored*. In this case, the *failure time* of the bridge is less than or equal to the age. By successively defining "failure" as the performance states $i = 2, 3, \ldots, N$ the time to reach each state (from state 1) can be derived using the procedure discussed above.

4.2 Holding time

Next is to determine the holding time distributions from knowledge of the distributions of T_i and T_j. Consider a difference of two arbitrary random variables $Z = X - Y$, the PDF of Z is given by

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-z)dx \quad (15)$$

$$\text{or} \quad \int_{-\infty}^{\infty} f_{X,Y}(y+z, y)dy \quad (16)$$

The joint distribution of X and Y is given by

$$f_{X,Y}(x,y) = f_Y(y) \cdot f_{X|Y}(x,y) \quad (17)$$

in which $f_{X|Y}(x|y)$ is the conditional distribution of X given Y. The PDF of Z is thus given by

$$f_Z(z) = \int_z^{\infty} f_Y(x-z) \cdot f_{X|Y}(x, x-z)dx \quad (18)$$

Notice that the integration in Eq. (18) starts at z. This is because x must be greater than z in order that $f_Y(x\text{-}z)$ is positive-valued.

The object now is to derive an expression for the random variable Z from the PDFs of X and Y. It is noted from Eq. (18) that in order to do so information about the joint distribution of X and Y is needed. However, the records in most bridge agencies are often not long enough to provide reliable information about the joint distribution. An assumption has to be made. For simplicity it is assumed that the number of (real) transitions at an epoch of the bridge deterioration process follows a nonhomogeneous Poisson process (NHPP) quite similar to a Weibull process. By treating the *counting process* as a quasi-Weibull process the PDF of Z could be obtained.

Some brief discussions on the Weibull process are in order. A *Weibull process* is defined as a nonhomogeneous Poisson process (NHPP) with an intensity function equals to the Weibull hazard function (Engelhart 1978) For a Weibull process, the conditional occurrence time of the second event, given the occurrence time of the first event, follows a truncated Weibull distribution with truncation point at the first occurrence (Engelhart 1978). Accordingly, the conditional density function for a Weibull process is

$$f_{X|Y}(x|y) = \frac{\alpha\kappa x^{\kappa-1} \cdot \exp\left[-\alpha x^{\kappa}\right]}{\exp\left[-\alpha y^{\kappa}\right]}, \quad 0 < y < x < \infty \tag{19}$$

Consider the present problem where Y is Weibull distributed with parameters (α_1, κ_1) and X is Weibull distributed with parameters (α_2, κ_2). It is assumed that the counting process of the number of transitions follows a quasi-Weibull process with the conditional distribution $f_{X|Y}(x|y)$ given by

$$f_{X|Y}(x|y) = \frac{\alpha_2\kappa_2 x^{\kappa_2-1} \cdot \exp\left[-\alpha_2 x^{\kappa_2}\right]}{\exp\left[-\alpha_2 y^{\kappa_2}\right]}, \quad 0 < y < x < \infty \tag{20}$$

Eq. (20) is modified from Eq. (19). Using Eq. (18) the following expression is obtained for the PDF of Z:

$$f_Z(z) = \int_z^{\infty} \frac{\alpha_1\alpha_2\kappa_1\kappa_2 (x-z)^{\kappa_1-1} x^{\kappa_2-1} \exp\left[-\alpha_1 (x-z)^{\kappa_1}\right]}{\exp\left[-\alpha_2 (x-z)^{\kappa_2}\right]} \cdot \exp[-\alpha_2 x^{\kappa_2}]dx$$

$$= \alpha_1\alpha_2\kappa_1\kappa_2 \int_z^{\infty} (x-z)^{\kappa_1-1} x^{\kappa_2-1} \cdot \exp\left[-\alpha_1(x-z)^{\kappa_1} - \alpha_2[x^{\kappa_2} - (x-z)^{\kappa_2}]\right]dx \tag{21}$$

By letting $Y = T_i$ and $X = T_{i+1}$, the PDFs of the holding time, $h_{i+1}(t)$ for $i = 1, 2, \ldots N$ can be computed using Eq. (21).

5 RESULTS

5.1 Indiana Example

The 1991 National Bridge Inventory (NBI) data from the state of Indiana, U. S. had been analyzed using the methods described in Chapter 4. A condition rating system from 9 (state 1: "best" condition) to 3 (state 7: "worst" condition) had been used. Because of the mathematical complexity of the formulation, a computer software, *Mathematica* (Walfram 1991) had been used to solve some of the equations numerically.

TABLE 1 Interval Probabilities, $\Phi(m)$:

m = 1 year

0.923	0.077	0.000	0.000	0.000	0.000	0.000
0.000	0.983	0.017	0.000	0.000	0.000	0.000
0.000	0.000	0.984	0.016	0.000	0.000	0.000
0.000	0.000	0.000	0.981	0.019	0.000	0.000
0.000	0.000	0.000	0.000	0.975	0.025	0.000
0.000	0.000	0.000	0.000	0.000	0.990	0.010
0.000	0.000	0.000	0.000	0.000	0.000	1.000

m = 2 years

0.785	0.213	0.001	0.000	0.000	0.000	0.000
0.000	0.964	0.036	0.000	0.000	0.000	0.000
0.000	0.000	0.968	0.032	0.000	0.000	0.000
0.000	0.000	0.000	0.962	0.037	0.001	0.000
0.000	0.000	0.000	0.000	0.950	0.049	0.001
0.000	0.000	0.000	0.000	0.000	0.980	0.020
0.000	0.000	0.000	0.000	0.000	0.000	1.000

m = 3 years

0.627	0.368	0.005	0.000	0.000	0.000	0.000
0.000	0.942	0.057	0.001	0.000	0.000	0.000
0.000	0.000	0.951	0.048	0.001	0.000	0.000
0.000	0.000	0.000	0.943	0.055	0.002	0.000
0.000	0.000	0.000	0.000	0.925	0.074	0.001
0.000	0.000	0.000	0.000	0.000	0.969	0.031
0.000	0.000	0.000	0.000	0.000	0.000	1.000

m = 4 years

0.473	0.515	0.012	0.000	0.000	0.000	0.000
0.000	0.918	0.080	0.002	0.000	0.000	0.000
0.000	0.000	0.934	0.064	0.002	0.000	0.000
0.000	0.000	0.000	0.924	0.074	0.003	0.000
0.000	0.000	0.000	0.000	0.900	0.098	0.002
0.000	0.000	0.000	0.000	0.000	0.958	0.042
0.000	0.000	0.000	0.000	0.000	0.000	1.000

m = 5 years

0.340	0.638	0.022	0.000	0.000	0.000	0.000
0.000	0.892	0.105	0.003	0.000	0.000	0.000
0.000	0.000	0.916	0.081	0.003	0.000	0.000
0.000	0.000	0.000	0.903	0.092	0.005	0.000
0.000	0.000	0.000	0.000	0.875	0.122	0.003
0.000	0.000	0.000	0.000	0.000	0.947	0.053
0.000	0.000	0.000	0.000	0.000	0.000	1.000

Table 1 presents the interval probabilities for concrete bridges which are obtained using Eq. (11) and Eq. (12). Only five years' result is presented. For illustration suppose that the bridge has just entered condition 8 (state $i = 2$) the probability that it is in state 7 (state $j = 3$) in 5 years' time is given by $\psi_{23}(5) = 0.105$.

Future bridge needs could be estimated by using Eq. (10). For illustration, suppose that out of 1,000 bridges in the network, the number of bridges in each category of the condition ratings from 9 to 3 is 80, 100, 220, 200, 300, 80 and 20. By interpreting the percentages of bridges in each condition category as the probability measures, this information can be represented as

$$\pi(0) = \{0.08 \quad 0.10 \quad 0.22 \quad 0.2 \quad 0.3 \quad 0.08 \quad 0.02\}$$

The number of bridges in each category after 5 years is given by

$$\pi(5) = \pi(0).\,\Psi(5)$$

$$= \{0.08 \quad 0.10 \quad 0.22 \quad 0.2 \quad 0.3 \quad 0.08 \quad 0.02\} \times$$

$$\begin{bmatrix} 0.340 & 0.638 & 0.022 & 0 & 0 & 0 & 0 \\ 0 & 0.892 & 0.105 & 0.003 & 0 & 0 & 0 \\ 0 & 0 & 0.916 & 0.081 & 0.003 & 0 & 0 \\ 0 & 0 & 0 & 0.903 & 0.092 & 0.005 & 0 \\ 0 & 0 & 0 & 0 & 0.875 & 0.122 & 0.003 \\ 0 & 0 & 0 & 0 & 0 & 0.947 & 0.053 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \{0.027 \quad 0.140 \quad 0.214 \quad 0.199 \quad 0.282 \quad 0.113 \quad 0.025\}$$

The results show that in 5 years' time the number of bridges in condition ratings 9, 8, ..., 3 is 27, 140, 214, 199, 282, 113 and 25; respectively.

The interval probability matrix can also be used to find the expected condition rating as given by

$$E[\xi] = \pi(0)\ \Psi(m)\ \{\ \xi_i\ \} \qquad (22)$$

where $\{\ \xi_i\ \}$ is the state space $\{\ 9, 8, 7, 6, 5, 4, 3\ \}^T$, of superstructure condition ratings. This relationship is used to determine the expected condition rating at each point in time.

5.2 Comparisons of the Semi-Markov Deterioration Model with Previous Studies

One way to compare the proposed semi-Markov deterioration model with previous studies is to compare the "average condition versus time" relationship produced by each study. This relationship is simply called regression function here. The regression functions for semi-Markov model were computed by using Eq. (22) to compute the average condition rating at each time point m. The regression functions thus obtained were compared with three regression models estimated using ordinary least square method.

All the three regression models were based on Indiana data collected around 1988 and 1989. Reconstructed bridges were not considered. In the first model developed by Jiang & Sinha (1990), the data was fit with a third-order polynomial function. The regression functions were forced to pass through condition 9 at time $t = 0$. The second regression model was produced by this author using Jiang et al's approach on Indiana's 1991 NBI data. However, instead of specifying the order of polynomial function to fit the data, the 'Stepwise' procedure of SAS (1993) was used to pick the variables for the model which are significant at 0.15 level; and thus deciding the order of the polynomial function. This way, the forms of the regression functions were not predetermined but dependent on the data. This model was developed merely as a "control" and will be so identified in the discussions that follow. The third regression model was obtained by Veshosky et al (1994). In this model, the data was fit with linear combination of logorithmic time and ADT.

Comparisons of all three regression models (ordinary least square method) with that derived from the semi-Markov model for steel bridges are shown in Fig. 2. Because each of the three regression functions (derived using ordinary least square method) could have been also used to produce the transition matrix for the Markov chain model (Jiang & Sinha 1990, Cesare et al 1992), Fig. 2 is essentially a comparison between the Markov chain model and the semi-Markov model.

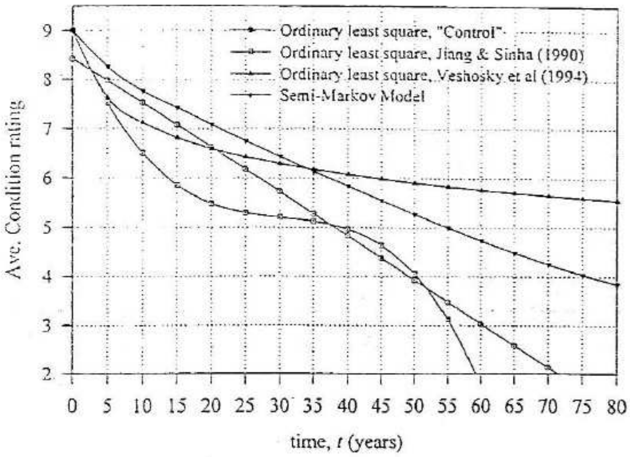

Fig.2 Steel Bridges of Indiana
Comparisons of various regression functions

Jiang and Sinha's model maintain the "flat-S" shape thanks to the specification for a third-order polynomial function. In the case where the order of the polynomial function was not specified (in "control" model) a linear regression function was obtained.

In general, Jiang and Sinha's models tend to predict shorter lives than by any other models; while Veshosky's model is just the opposite. This phenomenon is largely due to the distributional forms which each approach has presumed. A third-order polynomial function (assumed in Jiang and Sinha's model) would bring the curve down as the time increases. Besides the advantages of a semi-Markov deterioration model over existing models in concepts and formulation efficiency, the regression functions derived from the semi-Markov model appear to be "most natural" because 1) they do not presume any functional forms; 2) they are derived from a stochastic model and 3) they remain within the two extremes of cases found in Jiang et al's model and Veshosky et al's model.

6 CONCLUSIONS

This paper has outlined the procedure to model bridge deterioration as a semi-Markov process. Application of the procedure on Indiana data and comparisons of the results with previous deterioration studies show that the proposed methodology is reasonable. This procedure can also be used for modeling load rating deterioration. Its application is indeed not restricted to bridges but other infrastructure facilities as well.

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